Correction Mat 103
Examen janvier 2021

Exercice 1 1) $\mathcal{D}_f = \mathbb{R}$ $\quad \mathcal{D}_g = \{x \in \mathbb{R}, x \neq 0 \text{ et } 3x+1>0\}$

$= \{x \neq 0 \text{ et } x > -\frac{1}{3}\} =]-\frac{1}{3}; \infty[\setminus \{0\}$

$f'(x) = (2x + 3x^2) e^{3x+1}$

$g'(x) = \frac{3}{x(3x+1)} - \frac{1}{x^2} \ln(3x+1)$.

2) $f'(x) = 6\cos(2x)$ $\quad f''(x) = -12\sin(2x)$.

3) $\frac{\partial f}{\partial x} = (1+3y)e^{3xy}$ $\quad \frac{\partial f}{\partial y} = 3x^2 e^{3xy}$

Exercice 2 1) $F(x) = \frac{5}{13}x^{13} + \frac{\sin(3x)}{3} - \frac{1}{x}$ primitive de f

$G(x) = 3\ln|x^2+1| = 3\ln(x^2+1)$ primitive de g.

2) $\int_0^1 t e^{2t^2-1} dt = \left[\frac{e^{2t^2-1}}{4}\right]_0^1 = \frac{1}{4}(e-1)$

3) $F'(x) = \frac{1}{\sqrt{x}} + (x-1)\left(-\frac{1}{2}\right)x^{-3/2} = \frac{1}{x\sqrt{x}}\left(x - \frac{1}{2}(x-1)\right) = \frac{1}{x\sqrt{x}}\left(\frac{x}{2} + \frac{1}{2}\right)$

$= f(x)$

Oui.

Exercice 3 1) $\det A = -1 \neq 0$ donc A inversible

$A^{-1} = \frac{1}{-1}\begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}$.

$\det B = 0$ B non inversible.

CORRECTION 2

Exercice 3 (suite)

2) $A \in M_{32}$ $B \in M_{23}$ Donc AB possible
BA aussi

$AB = \begin{pmatrix} -3 & 0 & 1 \\ 19 & 5 & 12 \\ 5 & 1 & 2 \end{pmatrix} \in M_{33}$ $BA = \begin{pmatrix} 4 & 4 \\ 7 & 0 \end{pmatrix} \in M_{22}$

3) $A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ $A^2 = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 8 \\ -4 & 7 \end{pmatrix}$ $-2A = \begin{pmatrix} -2 & -4 \\ 2 & -6 \end{pmatrix}$

$A^2 - 2A + I = \begin{pmatrix} -2 & 4 \\ -2 & 2 \end{pmatrix}$ $(A - I)^2 = \begin{pmatrix} 0 & 2 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 4 \\ -2 & 2 \end{pmatrix}$

égalité.

Exercice 4 $u_0 = 1$ $u_{n+1} = \dfrac{u_n + 3}{2}$

1) $u_1 = 2$ $u_2 = \dfrac{5}{2}$

2) $v_{n+1} = 3 - \dfrac{u_n + 3}{2} = \dfrac{6 - u_n - 3}{2} = \dfrac{3 - u_n}{2} = \dfrac{1}{2} v_n$.

Donc $(v_n)_n$ est géom. de raison $\dfrac{1}{2}$.

$v_n = v_0 \dfrac{1}{2^n} = \dfrac{2}{2^n} = \dfrac{1}{2^{n-1}}$. D'où $u_n = 3 - \dfrac{1}{2^{n-1}}$.